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Letter

On relativistic approaches to the pion self-energy in nuclear matter

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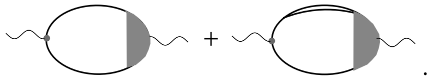
Abstract. We argue that, in contrast to the non-relativistic approach, a relativistic evaluation of the nucleon–hole and delta-isobar-nucleon–hole contributions to the pion self-energy incorporates the *s*-wave scattering, whose magnitude within the RPA is in conflict with the near-threshold behavior imposed by chiral symmetry. As a result, a relativistic approach to the pion self-energy in isospin-symmetric nuclear matter, containing only these diagrams, does not satisfy the known experimental results on the near-threshold behavior of the π-nucleon (forward) scattering amplitude.

PACS. 24.10.Cn Many-body theory – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 21.65.+f Nuclear matter – 25.70.-z Low and intermediate energy heavy-ion reactions

Originated from experiments with relativistic heavy-ion collisions, considerable efforts from many theoretical groups were made in relativistic approaches to the pion self-energy in isospin-symmetric nuclear matter (see, *e.g.*, [1–5] and references therein). Basically, such calculations, involving relativistic kinematics, are restricted to the contributions from nucleon particle–hole (ph) and Δ-isobar-nucleon–hole (Δh) excitations in the medium, as given by the following diagrams:

$\widetilde{\Pi} =$  .

Here the Δ-isobar is shown by a double line. The shaded effective vertices for the pion interaction with nucleons and deltas take into account the correlations in the medium. These vertices are irreducible with respect to pion lines.

As has been pointed out by many authors, such calculations yield a pion self-energy $\tilde{\Pi}(\omega, k)$, which, in the low-density limit, does not reproduce exactly the pion self-energy obtained from the non-relativistic reduction of the pion-nucleon and pion-delta Lagrangian. The purpose of this Letter is to illuminate the reason of this discrepancy, by showing that a relativistic approach containing only the above diagrams does not describe the pion self-energy appropriately.

In the following, we employ the widely used pseudovector interaction of pions with nucleons and deltas. The corresponding Lagrangian density can be written in the following form:

$$\mathcal{L}_{\text{int}} = \frac{f}{m_\pi}\bar{\psi}_N\gamma^\mu\gamma_5\boldsymbol{\tau}\psi_N\partial_\mu\boldsymbol{\varphi} + \frac{f_\Delta}{m_\pi}\bar{\psi}^\mu_\Delta\mathbf{T}^+\psi_N\partial_\mu\boldsymbol{\varphi} + \frac{f_\Delta}{m_\pi}\bar{\psi}_N\mathbf{T}\psi^\mu_\Delta\partial_\mu\boldsymbol{\varphi}. \tag{1}$$

Here $\boldsymbol{\varphi}$ is the pseudoscalar isovector pion field, m_π is the bare pion mass, and $f = 0.988$ is the pion-nucleon coupling constant. The excitation of the Δ in pion-nucleon scattering is described by the last two terms in the Lagrangian with the Chew-Low value of the coupling constant, $f_{\pi N\Delta} = 2f$. The nucleon field is denoted by ψ_N, and ψ_Δ stands for the Rarita-Schwinger spinor of the Δ-baryon. Here and below, we denote by $\boldsymbol{\tau}$ the isospin-(1/2) operators, which act on the isobaric doublet ψ of the nucleon field. The Δ-baryon is an isospin-(3/2) particle represented by a quartet of four states. $\mathbf{T}$ are the isospin transition operators between the isospin-(1/2) and -(3/2) fields. We use the system of units $\hbar = c = 1$.

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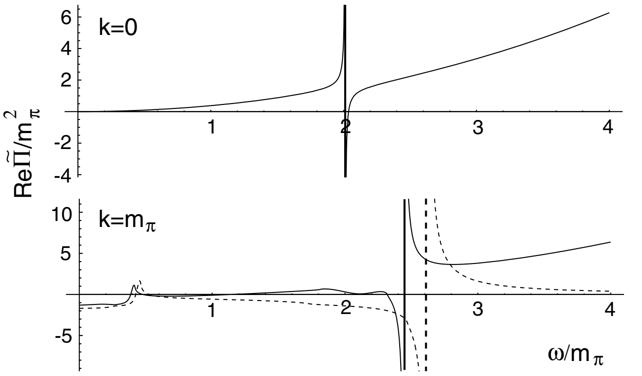


Fig. 1. Pion self-energy in isosymmetric nuclear matter at saturation density $n = n_0$. The effective nucleon mass is taken to be $M^* = 0.8M$. The solid line is obtained in the relativistic approach. The dashed line corresponds to a standard non-relativistic calculation. For $k = 0$, the non-relativistic counterpart of $\mathrm{Re}\tilde{\Pi}$ vanishes identically.

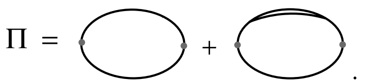
The non-relativistic reduction of the pion-nucleon and pion-delta coupling, given by eq. (1), leads to an effective interaction Hamiltonian of the form (see, *e.g.*, [6,7]):

$$\mathcal{H}_{\mathrm{int}} = \frac{f}{m_\pi}(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla})(\boldsymbol{\tau}\cdot\boldsymbol{\varphi}) + \frac{f_\Delta}{m_\pi}\left(\mathbf{S}^+\cdot\boldsymbol{\nabla}\right)\left(\mathbf{T}^+\cdot\boldsymbol{\varphi}\right) + \mathrm{h.c.}, \tag{2}$$

where $\boldsymbol{\sigma}$ are the Pauli matrices, and $\mathbf{S}^+$ are the transition spin operators connecting spin-(1/2) and -(3/2) states.

To show explicitly the above-mentioned problem, we perform both a relativistic and a non-relativistic calculation of the pion self-energy in a simple model, where the NN, $N\Delta$, and $\Delta\Delta$ correlations are simulated by phenomenological contact interactions with three Landau-Migdal parameters, g'_{NN}, $g'_{N\Delta}$, $g'_{\Delta\Delta}$. (For details of the calculation see [8].) Modern experiments and theoretical estimates [9–11] point out that $g'_{N\Delta}$ must be essentially smaller than g'_{NN} and $g'_{\Delta\Delta}$. The most recent analysis, reported in [12], suggests $g'_{NN} = 0.6$, $g'_{N\Delta} = 0.24 \pm 0.10$, $g'_{\Delta\Delta} = 0.6$. While we do not discuss possible deviations from this set of Landau-Migdal parameters, let us investigate the behavior of the pion self-energy in this case.

In fig. 1, the solid line represents the pion self-energy as obtained in relativistic calculations. For comparison, the dashed line shows the pion self-energy as calculated in the non-relativistic approach. As one can see, even for $k = 0$, we obtain a large discrepancy. In this case the non-relativistic counterpart of the pion self-energy vanishes identically, $\mathrm{Re}\tilde{\Pi}^{\mathrm{nr}}(\omega, k \to 0) = 0$, while the relativistic calculation results in an increase of the pion self-energy along with ω. One can easily find that the discrepancy arises even at the lowest-order level:



Consider, for example, the particle-hole contribution, as given by the first one-loop diagram. The relativistic evaluation yields (see, *e.g.*, [2]):

$$\mathrm{Re}\Pi_{ph}(\omega, k) = \frac{f^2}{\pi^2}\frac{K_\mu K^\mu M^{*2}}{m_\pi^2 k} \times \int_0^{p_{\mathrm{F}}} \frac{\mathrm{d}pp}{\varepsilon} \ln\left|\frac{\left(K_\mu K^\mu - 2kp\right)^2 - 4\omega^2\varepsilon^2}{\left(K_\mu K^\mu + 2kp\right)^2 - 4\omega^2\varepsilon^2}\right|, \tag{3}$$

where M^* is the effective nucleon mass, $\varepsilon^2 = M^{*2} + p^2$, p_{F} is the nucleon Fermi momentum, and $K^\mu = (\omega, \mathbf{k})$ is the pion four-momentum.

It is instructive to analyze the low-density limit of this expression in order to compare with the known non-relativistic form. At low density of nucleons, $p_{\mathrm{F}}/M^* \ll 1$, one has $\varepsilon(p) \simeq M^*$. With this replacement, the integration can be performed to give

$$\mathrm{Re}\Pi_{ph}(\omega, k) = \frac{4f^2}{m_\pi^2}\left(\omega^2 - k^2\right) \times \left(\Phi_0(\omega, k; p_{\mathrm{F}}) + \Phi_0(-\omega, k; p_{\mathrm{F}})\right), \tag{4}$$

where

$$\Phi_0(\omega, k; p_{\mathrm{F}}) = \frac{1}{4\pi^2}\frac{M^{*3}}{k^3} \times \left(\frac{1}{2}\left(a^2 - k^2V_{\mathrm{F}}^2\right)\ln\frac{a + kV_{\mathrm{F}}}{a - kV_{\mathrm{F}}} - akV_{\mathrm{F}}\right) \tag{5}$$

is the Migdal function, with

$$a = \omega + \frac{\omega^2 - k^2}{2M^*}, \qquad V_{\mathrm{F}} = p_{\mathrm{F}}/M^*. \tag{6}$$

This non-relativistic limit to the lowest-order particle-hole self-energy has been obtained from relativistic kinematics. As given by eq. (4), for $\omega \ll 2M^*$ and in the limiting case of $k \to 0$, we have

$$\mathrm{Re}\Pi_{ph}(\omega, k \to 0) = \frac{f^2}{M^* m_\pi^2} n\omega^2 - \frac{f^2}{m_\pi^2} nk^2\left(\frac{1}{k^2/2M^* - \omega} + \frac{1}{k^2/2M^* + \omega}\right) \tag{7}$$

with

$$n = \frac{2p_{\mathrm{F}}^3}{3\pi^2}$$

being the number density of nucleons for isosymmetric nuclear matter. The corresponding relativistic calculation of the lowest-order Δh loop gives an expression, which, in the low-density limit and $\omega \ll 2M^*$, takes the following form [8]:

$$\mathrm{Re}\Pi_{\Delta h}(\omega, k \to 0) = \frac{8f_\Delta^2}{9m_\pi^2}\frac{M^* + M_\Delta^*}{M_\Delta^{*2}} n\omega^2 + \frac{8f_\Delta^2}{9m_\pi^2}\frac{M_\Delta^* - M^*}{\omega^2 - \left(M_\Delta^* - M^*\right)^2} nk^2\,. \tag{8}$$

A comparison of eq. (7) and eq. (8) with the non-relativistic form of the lowest-order pion self-energy [7]

$$\mathrm{Re}\Pi^{\mathrm{nr}}\left(\omega, k\to 0\right) = -\frac{f^2}{m_\pi^2}nk^2\left(\frac{1}{k^2/2M^*-\omega}\right.$$
$$\left.+\frac{1}{k^2/2M^*+\omega}\right)+\frac{8f_\Delta^2}{9m_\pi^2}\frac{M_\Delta^*-M^*}{\omega^2-\left(M_\Delta^*-M^*\right)^2}nk^2 \quad (9)$$

shows that the relativistic evaluation results in additional contributions, which do not vanish when $k\to 0$. In fact, one finds

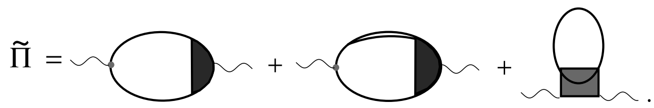
$$\mathrm{Re}\Pi_{ph}(\omega,k\to 0)+\mathrm{Re}\Pi_{\Delta h}(\omega,k\to 0)-\mathrm{Re}\Pi^{\mathrm{nr}}(\omega,k\to 0)=$$
$$\left(\frac{f^2}{M^*m_\pi^2}+\frac{8f_\Delta^2}{9m_\pi^2}\frac{M^*+M_\Delta^*}{M_\Delta^{*2}}\right)n\omega^2 . \quad (10)$$

The discrepancy appears at the order $1/M^*$ and therefore should be suppressed as compared to the $O(1)$ contributions. However, we recall that the p-wave contribution vanishes identically when $k\to 0$. Therefore, even at normal density, $n=n_0$, the above s-wave contribution results in an increase, up to 20%, of the near-threshold effective pion mass, defined as $m_\pi^* = \sqrt{m_\pi^2+\tilde{\Pi}\left(m_\pi^*,0\right)}$. This contribution should be crucial in the case of large transferred energy, as typical for experiments with relativistic heavy-ion collisions.

To explain the origin of these terms, we recall that the pion self-energy represents the forward-scattering amplitude of the pion in the medium. In the non-relativistic theory, the above ph and Δh diagrams, taking also into account the correlations in the medium, are known to reproduce well the (forward) p-scattering amplitude in the isospin-symmetric nuclear matter, while the s-scattering contribution is known to be small. Due to the non-relativistic reduction of πNN interaction, as given by eq. (2), the s-wave scattering gives no contribution to the ph and Δh diagrams.

However, the relativistic form of the pion-nucleon and pion-delta interactions, as given by eq. (1), causes an s-wave contribution to the above diagrams. Consider, for example, the πNN interaction. At the pion four-momentum $K^\mu = (\omega, \mathbf{k}=0)$, these couplings involve only the time component of the currents. In the low-density limit, $p_{\mathrm{F}}/M^* \ll 1$, the matrix element $\langle N(p')|\,\bar{\psi}_N\gamma^0\gamma_5\boldsymbol{\tau}\psi_N\,|N(p)\rangle$ is proportional to $\boldsymbol{\sigma}\cdot\left(\mathbf{p}+\mathbf{p}'\right)/2M^*$, and the non-relativistic reduction, eq. (2), of the πNN interaction implies that the part of the scattering amplitude generated by $\mathcal{L}_{\mathrm{int}}$ at the second order vanishes for nucleons at rest. However, this is not the case, if the time component of the interaction is relativistically incorporated into the calculation of the ph and Δh diagrams. Integration over the nucleon momentum results in a contribution, proportional to ω^2, as given by eq. (10).

Thus, in contrast to the non-relativistic approach, a relativistic evaluation of the ph and Δh contributions to the pion self-energy incorporates the s-wave scattering. This means that a covariant relativistic evaluation of the pion self-energy cannot be restricted only to the ph and Δh diagrams. A correct calculation of the (forward) s-wave amplitude actually requires the inclusion of many more complicated diagrams, since the s-scattering is caused mostly by the short-distance interactions, with a scale $r_0 \sim M^{-1}$. In this case the pion self-energy should be represented as the sum of three graphs:



Here the third diagram represents the contribution to the (forward) s-wave amplitude caused by the above short-distance interactions in the medium. The rectangular block includes many diagrams which have no parts connected either by the particle-hole or Δ-isobar and nucleon hole lines. All the diagrams of this type are characterized by large momenta in the intermediate states, of the order M, and thus contribute to the pion self-energy at the order M^{-1}. When these diagrams are included, one can expect a very strong cancellation of the s-wave contribution to the pion self-energy. Indeed, for example, at the threshold $\omega = m_\pi$, from eq. (7) and eq. (8) we obtain the forward s-scattering amplitude as

$$\frac{1}{n}\mathrm{Re}\Pi\left(m_\pi,0\right)=\left(\frac{f^2}{M^*}+\frac{8f_\Delta^2}{9}\frac{M^*+M_\Delta^*}{M_\Delta^{*2}}\right)\simeq 1.3\,\mathrm{fm}. \quad (11)$$

The correlation effects, as well as a reasonable variation of the effective nucleon (and delta) mass, do not change the order of magnitude of this result. When the short-range correlations are taken into account we obtain

$$\frac{1}{n}\mathrm{Re}\tilde{\Pi}\left(m_\pi,0\right)\simeq 1.11\,\mathrm{fm}. \quad (12)$$

Chiral symmetry, however, imposes strong constraints on the near-threshold behavior of this isospin-even amplitude [13] and it is known experimentally [14,15] to be much smaller, as compared to that given by eq. (11) and eq. (12).

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